SCRS/2006/079 Col. Vol. Sci. Pap. ICCAT, 60(3): 1014-1026 (2007)

ARE THE GROWTH CURVES CURRENTLY USED FOR ATLANTIC BLUEFIN TUNA STATISTICALLY DIFFERENT?

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SUMMARY

*Two different growth curves are used currently for the eastern and western stocks of Atlantic bluefin tuna (*Thunnus thynnus*). These curves were estimated after pulling together various data sets that were collected with several methods: modal analysis of length frequencies, direct ageing from spines and tagging studies. In this study, we reconstructed datasets that were as close as possible to the original ones that were used to estimate the growth curves. We carried out statistical tests based on maximum likelihood theory to examine if different sets of data supported the adoption of different growth curves. We used two alternative error structures to guard against the possibility that this assumption would unduly influence the results. The results obtained indicate that the different reconstructed data sets (between and within stocks) support the estimation of different growth curves. Some of these results may be partly due to differences in the size and age ranges of the different datasets. We conclude that age validation research must be intensified in order to resolve these issues.*

RÉSUMÉ

*Deux courbes de croissances différentes sont actuellement utilisées pour les stocks de l'est et de l'ouest du thon rouge de l'Atlantique (*Thunnus thynnus*). Ces courbes ont été estimées après avoir regroupé plusieurs jeux de données collectés par diverses méthodes : analyse modale des fréquences de tailles, détermination directe de l'âge d'après les épines et études de marquage. Dans la présente étude, nous avons reconstruit les jeux de données qui étaient aussi proches que possible des originaux utilisés pour estimer les courbes de croissance. Nous avons procédé à des tests statistiques basés sur la théorie de vraisemblance maximale pour déterminer si différents jeux de données supportaient l'adoption de courbes de croissance distinctes. Nous avons employé deux structures d'erreur alternatives afin d'éviter la possibilité que ce postulat n'influence trop les résultats. Les résultats obtenus indiquent que les différents jeux de données reconstruits (entre les stocks et au sein d'entre eux) étayent l'estimation de courbes de croissance différentes. Certains de ces résultats pourraient être dus, en partie, à des différences dans les gammes de tailles et d'âges des différents jeux de données. Notre conclusion est que les travaux de recherche portant sur la validation de l'âge doivent être intensifiés en vue de résoudre ces questions.*

RESUMEN

*Actualmente se utilizan dos curvas de crecimiento diferentes para los stocks de atún rojo del Atlántico occidental y oriental (*Thunnus thynnus*). Estas curvas fueron estimadas tras reunir varios conjuntos de datos que fueron recopilados con diversos métodos: análisis modal de las frecuencias de talla, determinación directa de la edad a partir de espinas y estudios de marcado. En este estudio hemos reconstruido conjuntos de datos que estaban lo más cercanos posible a los originales que se utilizaron para estimar las curvas de crecimiento. Hemos llevado a cabo pruebas estadísticas basadas en la teoría de máxima verosimilitud para examinar si diferentes conjuntos de datos respaldaban la adopción de curvas de crecimiento diferentes. Hemos utilizado dos estructuras de error alternativas para evitar la posibilidad de que este supuesto influya de forma excesiva en los resultados. Los resultados obtenidos indican que los diferentes conjuntos de datos reconstruidos (entre y dentro de los stocks) respaldan la*

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estimación de diferentes curvas de crecimiento. Algunos de estos resultados pueden deberse, en parte, a diferencias en los rangos de edad y talla de los distintos conjuntos de datos. Hemos concluido que para resolver estos temas debe intensificarse la investigación sobre la validación de la edad.

KEYWORDS

Growth curves, age determination, tagging

1. Introduction

The growth curves currently used by the ICCAT Standing Committee on Research and Statistics (SCRS) in bluefin tuna assessments were estimated in the early 1990s independently for each bluefin tuna stock, by Cort (1991) for the eastern Atlantic and Mediterranean and by Turner *et al*. (1991) for the western Atlantic stock. The latter work was updated by Turner and Restrepo (1994). There are differences between these two growth curves, especially for older fish, with an increasing divergence that reaches one year of difference from age 8 (**Figure 1**). The differences in growth between both stocks need to be reviewed to determine if discrepancies are statistically significant. These differences have implications not only in the age structure of each stock, but also in the construction of mixing models.

The objective of this paper is to compare the two growth curves statistically, via likelihood ratio tests.

2. Materials and methods

2.1 Data

It was not possible to recover the exact data sets used by Cort (1990; 1991) and Turner *et al*. (1991), so attempts were made to produce data sets that were as close as possible, following the descriptions given in those papers.

Cort datasets

Cort (1991) used primarily two data sets: length frequency distributions for small fish caught in the Cantabrian Sea by the Spanish baitboat fishery, and an age-length key (ALK) based on spine sections direct ageing from larger fish caught by tuna traps in the southern Atlantic coast of Spain. Cort (1990; 1991) adjusted an equation to monthly modal length distribution data values, corresponding to 1 to 5 year-old specimens. With this function, the lengths were estimated for ages 1 to 7 in the middle of the year. The month of June was selected because the age-length key was also obtained from the trap specimens caught in that month; this ALK was used to estimate length for ages 8 to 15. For the present study, two reconstructed data sets were used:

Dataset C1: Dataset with the original mean values from the modal length-frequency data, plus the mean length at age from the ALK (this dataset is composed of 28 mean values ranging from 56 to 247 cm in straight fork length -SFL-, and ages 1.1 to 15).

Dataset C2(a): Modal length distributions were reconstructed with the same mean and standard deviation than those employed by Cort (1990) (the total sample size in this data set was 210, ranging from ages 1.1 to 5.25, and from 55 to 149 cm SFL).

Dataset C2(b): The ALK in Table 7 of Cort (1991) was used (here, lengths were taken as the midpoints of the reported size intervals; the total sample size in this data set was 167, ranging from ages 9 to 15, and from 170 to 267 cm SFL).

Turner et al. datasets

Dataset T1(a): Turner *et al*. (1991) and Turner and Restrepo (1994) used a conventional tag-recapture database that contained releases made by USA and Canadian scientists up to 1990. Turner *et al*. (1991) explain several quality control procedures applied to cull the dataset (e.g., using only records with times at liberty greater than 2 months; using only fish with measured size at release and recapture, etc.). It was not possible to recover the same tagging database used by these authors. Instead, an extraction of data up to 1990 from the current database maintained by the ICCAT Secretariat was subjected to the same quality control procedures as in Turner *et al*. (1991) in an attempt to approximate the same dataset as much as possible. In addition, only records released and recaptured west of 45ºW were used. The resulting data set consisted of 816 fish, which is less than the 908 fish reported by Turner *et al*. (1991) having fork length measurements at both release and recapture times.

Dataset T1(b): A second source of data used by Turner *et al*. (1991) consisted of age-length observations derived from the midpoints of length-frequency distributions. These 103 readings, reported in Table 3 of Turner *et al*. (1991), were used in this paper (age ranging from 1.04 to 3.42 and from 51 to 102 cm SFL).

2.2 Likelihood estimation

Kirkwood (1983) provides a method to estimate the joint likelihood function of tag-recapture and age-length data sets as a function of the growth curve parameters. Turner *et al*. (1991) and Turner and Restrepo (1994) followed this approach to estimate the growth curves for the western stock. We used this approach as the basis for our analyses.

The method of Kirkwood (1983) uses age (or time at liberty, for tagging data) as the dependent variable. However, it is more common in the fisheries literature to model size (or change in size) as the dependent variable. We therefore also examined models with this option, following an approach recommended by Hampton (1991), based on Kirkwood and Somers (1984).

It is well known that the asymptotic size and the growth rate from the von Bertalanffy growth equation are highly and negatively correlated. Typically, when the data are restricted to small or large sizes (as happens with the datasets examined by us), one can obtain estimates of these two parameters that are clearly unrealistic. For this reason, we also report values of the product $\omega = L_\infty K$ which is less sensitive to the range of data (Quinn and Deriso, 1999), for ease of comparison.

2.2.1 Age (time at liberty) as the dependent variable

This is the approach in Kirkwoord (1983). Let

L_∞, K, t_0 = parameters of the von Bertalanffy growth curve,

δt_i = time at liberty,

δl_i = change in length between release and recapture

R_i = release length.

Assuming a normal error structure, the corresponding log likelihoods for a given set of growth parameters and variances are as follows (the subscripts 1 and 2 indicate tagging data and age-length observations, respectively):

$$\Phi_1 = -\frac{n_1}{2}\ln(2\pi\sigma_1^2) - \frac{1}{2\sigma_1^2}\sum_{i=1}^{n_1}\left[\delta t_i + \frac{1}{K}\ln\left(1 - \frac{\delta l_i}{L_\infty - R_i}\right)\right]^2$$

$$\Phi_2 = -\frac{n_2}{2}\ln(2\pi\sigma_2^2) - \frac{1}{2\sigma_2^2}\sum_{i=1}^{n_2}\left[t_i - t_0 + \frac{1}{K}\ln\left(1 - \frac{l_i}{L_\infty}\right)\right]^2$$

When using only age-length data, maximum likelihood estimates of the growth parameters are obtained by finding the values that maximize Φ_2. When using both age-length and tagging data, the parameters are obtained by maximizing the joint likelihood $\Lambda = \Phi_1 + \Phi_2$.

2.2.2 Length (length increment) as the dependent variable

The model recommended by Hampton (1991) using tagging data for southern bluefin tuna is from Kirkwood and Somers (1984) which allows for individual growth variability by assuming that the asymptotic size is a random variable. Hampton (1991) modified it so as to also include a model error term. Below, we have extended the approach so that it can also be applied to length-age observations (see Φ_2):

Let

$\sigma^2_{L\infty}$ = variance of L_∞ ,

σ^2_m = variance for model error

$$\Phi_1 = -\sum_i \frac{\ln(2\pi(\sigma^2_{L\infty}(1-e^{-K\delta t_i})^2 + \sigma^2_{m,1})}{2} + \frac{(\delta l_i - ((L_\infty - R_i)(1-e^{-K\delta t_i}))^2}{2(\sigma^2_{L\infty}(1-e^{-K\delta t_i})^2 + \sigma^2_{m,1})}$$

$$\Phi 2 = -\sum_i \frac{\ln(2\pi(\sigma^2_{L\infty}(1-e^{-K(t_i - t_0)})^2 + \sigma^2_{m,2})}{2} + \frac{(l_i - (L_\infty(1-e^{-K(t_i - t_0)}))^2}{2(\sigma^2_{L\infty}(1-e^{-K(t_i - t_0)})^2 + \sigma^2_{m,2})}$$

When using two or more data sets to estimate the same growth parameters, we assume a common $\sigma^2_{L\infty}$ but different σ^2_m for each data set.

2.3 Likelihood ratio tests

Likelihood ratio tests can be used to compare sets of parameter estimates in models that are "nested" (Kirkwood, 1983). For example, one could test if the log likelihood obtained with *z* parameters is statistically significant from the likelihood obtained with a subset of the parameters *a* ($a<z$). This is accomplished by computing the statistic

$$T = -2({}^a\Lambda - {}^z\Lambda)$$

which follows a χ^2 distribution with (*z*-*a*) degrees of freedom.

Cerrato (1990) compared the performance of various approaches for comparing growth curves. He found likelihood ratio tests to perform generally much better than the other options he examined.

3. Results

3.1 Comparison between original growth curves and reconstructed ones

We first compared how the growth curves derived from the reconstructed datasets compared to the original growth curve estimates (**Figure 2**).

For the eastern stock, the Cort (1991) growth curve and that obtained from dataset C1 are practically indistinguishable (**Figures 2a** and **2b**). This is not surprising as only the mean lengths at age were used for estimation in both cases; only the estimation method (maximum likelihood in this paper vs least squares in the Cort, 1991, paper) differed slightly. The curve obtained with Dataset C2 (a and b) predicts somewhat larger sizes at old ages when using time as the dependent variable (**Figure 2a**), but is very similar when modeling length as the dependent variable (**Figure 2b**). This suggests that slightly different growth curves are obtained depending on whether the highly aggregated dataset (C1) or the disaggregated one (C2) are used, and depending on the assumed model error structure.

For the western stock, the curve estimated by Turner and Restrepo (1994) differs slightly from that which we attempted to reconstruct with Datasets T1 (a and b) (**Figures 2a** and **2b**). Because the maximum likelihood estimation procedure of Kirkwood (1983) and the age-length data (T1b) were exactly the same in both studies, it can be concluded that the original tagging data in Turner and Restrepo (1994) and the reconstructed dataset (T1a) differ somewhat.

In conclusion, it is apparent that dataset T1(b) is not exactly the same as used by Turner and Restrepo (1994) and for that reason the results in the following sections should be viewed with caution. Nevertheless, the growth curves estimated with the original and reconstructed data seem reasonably close so as to allow useful comparisons.

3.2 Comparisons between and within datasets

To allow for the various tests to be conducted, several Cases were defined in terms of what datasets to use in estimating the growth parameters:

CASE	
1	Fit one growth curve to each data set
2	Fit one curve to datasets T1(a and b) combined
3	Fit one curve to datasets C2(a and b) combined
4	Fit one curve to datasets T1(a and b) and C1 combined
5	Fit one curve to datasets T1(a and b) and C2(a and b) combined
6	Fit one curve to sets T1(b) and C2(a) combined

The resulting parameter estimates and log-likelihood values are provided in **Table 1**.

Five tests were conducted, as follows:

Test 1: Compare fitting a single growth curve to datasets C1, T1a and T1b, against fitting one curve to C1 and another one to T1a and T1b. This addresses the question "do the datasets used by Cort (1991) and Turner and Restrepo (1994) support the use of different growth curves?"

Test 2: Compare fitting a single curve to datasets T1a, T1b, C2a and C2b, against fitting one growth curve to T1a and T1b and another one to C2a and C2b. This is similar to Test 1, above, except that it uses more disaggregated data available to Cort (1991).

Test 3: Compare fitting a single curve to T1a and T1b, against fitting one curve to each dataset. This addresses the question of compatibility between the two datasets used by Turner and Restrepo (1994).

Test 4: Compare fitting a single curve to C2a and C2b, against fitting one curve to each dataset. This addresses the question of compatibility between the two detailed datasets available to Cort (1991).

Test 5: Compare fitting a single curve to C2a and T1b, against fitting one curve to each dataset. This addresses the question of compatibility between the two datasets for small bluefin.

The results of the likelihood ratio tests are summarized in **Table 2**, with both model error structures that were used in this paper. All comparisons were highly significant, suggesting that fitting separate growth curves to each data set was statistically preferable to fitting curves to the combined datasets.

In the case of Test 5, we noticed that datasets C2a and T1b seemed to be offset by a constant age (see **Figure 3**). We considered the possibility that the difference between the two was attributable to the assignment of birth dates used by Cort (1991) for the eastern stock and by Turner and Restrepo (1994) for the western stock. Therefore, we subtracted 1.5 months from all the age observations in dataset T1b and repeated Test 5. However, the resulting T statistic was still highly significant.

4. Discussion

Ageing bluefin tuna for the entire length range of the population is difficult because it is a highly mobile species with a long lifespan, and has extensive geographical ranges making it hard to find or sample all the lengths from the same area or fishery. This fact makes it necessary to join data from different sources to complete the whole length distribution or year classes if one attempts to study the growth of this species, otherwise the results would be biased. Currently used growth curves for the western and eastern bluefin tuna stocks are based on data that were collected with several methods: modal analysis of length frequencies, direct ageing from spines and tagging studies (Cort 1991; Turner *et al*., 1991; Turner and Restrepo, 1994).

We have used likelihood ratio tests to examine the consistency between the various datasets in terms to giving support to using one or more growth curves. Our focus has been on making such tests, rather than on estimating new growth curves or validating the curves that were estimated by the above authors. An important caveat is that we have not been able to replicate exactly all of the datasets, although we are comfortable that we reproduced datasets that are reasonably close. We caution the potential reader who would be eager to pick and choose estimates of (L_∞, K, t_0) in **Table 1**, that doing so can be misleading. If anything, the estimates of ω could be compared because the ranges of sizes do not overlap much between the various datasets.

The results obtained indicate that, overall, the data available to Cort (1991) and Turner *et al*. (1991) and Turner and Restrepo (1994) do indeed support using different values of (L_∞, K, t_0) for the two stocks. These different parameter sets result in growth curves that are very similar for bluefin younger than age 10 (**Figure 1**). The largest differences in expected growth becomes more evident for old fish.

Closer examination of the available information indicates that there are also differences between the subsets of data available to these authors. Both Cort (1991) and Turner et. al. (1991) pulled together data for small bluefin and large bluefin, for apparent different reasons. In the case of Cort (1991), he pulled together data from small fish coming from modal length distributions (set C2a) and large fish coming from direct ageing of spines (set C2b) in order to obtain a broad range of age and size observations for estimating the growth parameters. In the case of Turner *et al*. (1991), the main dataset was from tagging (T1a) which included fish at liberty for over 15 years and length increments of up to 186 cm. But tag-recapture data alone only provide information for estimating (L_∞, K), so the authors had to use modal length frequency analysis (T1b) in order to include some data with which to estimate t_0.

Our results indicate that sets C2a and C2b support using different growth curves, and that T1a and T1b also support using different growth curves. These results are not entirely unexpected, particularly for the differences between the subsets of data used by Cort (1991) because they cover entirely different size ranges. This can be appreciated in **Figure 4**, where pairs of (L_∞, K) values estimated by bootstrapping are shown for individual fits to the different datasets. Differences between subsets could also be due to changes in the pattern of growth between juveniles and adults stages. Hearn and Polacheck, (2002) and Bayliff *et al*., (1991) stated that a growth rate with a discontinuity at a certain size may be more common that the smooth von Bertalanffy model of growth, and that more complex growth models should be considered.

In conclusion, our analyses indicate that there may be systematic differences between (and within) datasets that have been used to estimate the growth curves for the eastern and western bluefin stocks. Ultimately, the questions raised can only be resolved through validation.

Preliminary results of bomb radiocarbon ageing (Neilson and Campana, SCRS/2006/077) of large fish (>250 cm) caught in Canadian waters indicate that L_∞ may be overestimated as the fish are considerably older than would be predicted by either the Turner and Restrepo (1994) or the Cort (1991) growth equations. We recommend that such validation studies be intensified.

In addition, fifteen years have elapsed since the Cort (1991) and Turner *et al*. (1991) studies. During this time, more data have been accumulated for both stocks from tagging and from direct age readings. In addition, the possibility that growth rates have changed should be considered.. For these reasons, we also recommend that the bluefin species group consider revisiting the growth curves used for both stocks.

Acknowledgments

Geoff Kirkwood passed away earlier this year. He was a gentleman and a scientist with many very good ideas who will be missed.

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Table 1a. Estimates of the growth parameters and likelihood values obtained by fitting growth curves to different data sets (T1a, T1b, C1, C2a and C2b), modeling age (time at liberty) as the dependent variable. The six Cases and the datasets used are defined in the text.

	T1a	T1b	C1	C2a	C2b
CASE 1					
ω	31.4	30.4	30.1	27.6	13.2
L_∞	353.2	253.8	316.4	452.6	4768.6
K	0.089	0.120	0.095	0.061	0.003
t_0		-0.881	-0.988	-1.105	-5.383
σ	0.51	0.16	0.17	0.30	1.00
Φ	-608.19	44.02	10.55	-262.90	-237.57
CASE 2					
ω	30.2	30.2			
L_∞	362.3	362.3			
K	0.083	0.083			
t_0		-0.730			
σ	0.51	0.18			
Φ	-610.88	31.74			
CASE 3					
ω				31.3	31.3
L_∞				322.4	322.4
K				0.097	0.097
t_0				-0.856	-0.856
σ				0.31	1.31
Φ				-273.99	-281.87
CASE 4					
ω	30.0	30.0	30.0		
L_∞	354.4	354.4	354.4		
K	0.085	0.085	0.085		
t_0		-0.755	-0.755		
σ	0.51	0.17	0.56		
Φ	-615.91	33.95	-23.39		
CASE 5					
ω	30.8	30.8		30.8	30.8
L_∞	343.2	343.2		343.2	343.2
K	0.090	0.090		0.090	0.090
t_0		-0.844		-0.844	-0.844
σ	0.52	0.23		0.31	1.46
Φ	-616.98	5.34		-273.16	-300.08
CASE 6					
ω		28.1		28.1	
L_∞		442.2		442.2	
K		0.064		0.064	
t_0		-1.019		-1.019	
σ		0.23		0.31	
Φ		6.20		-267.87	

Table 1b. Estimates of the growth parameters and likelihood values obtained by fitting growth curves to different data sets (T1a, T1b, C1, C2a and C2b), modeling length (length increment) as the dependent variable. The six Cases and the datasets used are defined in the text.

	T1a	T1b	C1	C2a	C2b
CASE 1					
ω	25.5	27.3	29.7	23.1	40.2
L_∞	518.5	298.3	319.2	812.5	289.1
K	0.049	0.092	0.093	0.028	0.139
t_0		-1.115	-1.030	-1.590	1.196
$\sigma_{L\infty}$	77.33	0.02	0.02	48.91	13.63
σ_m	8.11	3.31	3.07	0.00	1.56
Φ	-2990.37	-269.40	-71.18	-3621.04	-629.51
CASE 2					
ω	27.3	27.3			
L_∞	384.5	384.5			
K	0.071	0.071			
t_0		-1.027			
$\sigma_{L\infty}$	19.05	19.05			
σ_m	9.37	1.45			
Φ	-3006.25	-280.50			
CASE 3					
ω				28.4	28.4
L_∞				334.4	334.4
K				0.085	0.085
t_0				-1.196	-1.196
$\sigma_{L\infty}$				20.013	20.013
σ_m				0.00	0.03
Φ				-3643.45	-638.46
CASE 4					
ω	28.0	28.0	28.0		
L_∞	353.7	353.7	353.7		
K	0.079	0.079	0.079		
t_0		-0.992	-0.992		
$\sigma_{L\infty}$	11.56	11.56	11.56		
σ_m	9.56	2.71	3.90		
Φ	-3010.48	-278.25	-86.11		
CASE 5					
ω	28.5	28.5		28.5	28.5
L_∞	336.0	336.0		336.0	336.0
K	0.085	0.085		0.085	0.085
t_0		-1.166		-1.166	-1.166
$\sigma_{L\infty}$	20.30	20.30		20.30	20.30
σ_m	9.27	3.83		0.00	0.12
Φ	-3005.40	-332.67		-3644.85	-640.10
CASE 6					
ω		23.0		23.0	
L_∞		868.1		868.1	
K		0.027		0.027	
t_0		-1.576		-1.576	
$\sigma_{L\infty}$		52.46		52.46	
σ_m		3.76		0.02	
Φ		-330.05		-3623.57	

Table 2. Results of the likelihood ratio tests (see text). Tests, cases and datasets used are defined in the text.

Based on age (time at liberty)

	Simple Model			Complex model					
Test	**Data/Case**	**# param**	**LL**	**Data/Case**	**# param**	**LL**	**T**	**d.f.**	**Prob.**
1	Case 4: C1,T1a,T1b	6	-605.35	Case 1: C1 Case 2: T1a,T1b	9	-568.59	73.52	3	7.52E-16
2	Case 5: T1a,T1b,C2a,C2b	7	-1184.88	Case 2: T1a,T1b Case 3: C2a,C2b	10	-1135	99.76	3	1.75E-21
3	Case 2: T1a,T1b	5	-579.14	Case 1: T1a Case 1: T1b	7	-564.17	29.94	2	3.15E-07
4	Case 3: C2a,C2b	5	-555.86	Case 1: C2a Case 1: C2b	8	-500.47	110.78	3	7.46E-24
5	Case 6: T1b,C2a	5	-261.67	Case 1: T1b Case 1: C2a	8	-218.88	85.58	3	1.95E-18

Based on length (length increment)

	Simple Model			Complex model					
Test	**Data/Case**	**# param**	**LL**	**Data/Case**	**# param**	**LL**	**T**	**d.f.**	**Prob.**
1	Case 4: C1,T1a,T1b	7	-3374.84	Case 1: C1 Case 2: T1a,T1b	11	-3357.93	33.82	4	8.11E-07
2	Case 5: T1a,T1b,C2a,C2b	8	-7623.02	Case 2: T1a,T1b Case 3: C2a,C2b	12	-7568.66	108.72	4	1.36E-22
3	Case 2: T1a,T1b	6	-3286.75	Case 1: T1a Case 1: T1b	9	-3259.77	53.96	3	1.14E-11
4	Case 3: C2a,C2b	6	-4281.91	Case 1: C2a Case 1: C2b	10	-4250.55	62.72	4	7.77E-13
5	Case 6: T1b,C2a	6	-3953.62	Case 1: T1b Case 1: C2a	10	-3890.44	126.36	4	2.34E-26

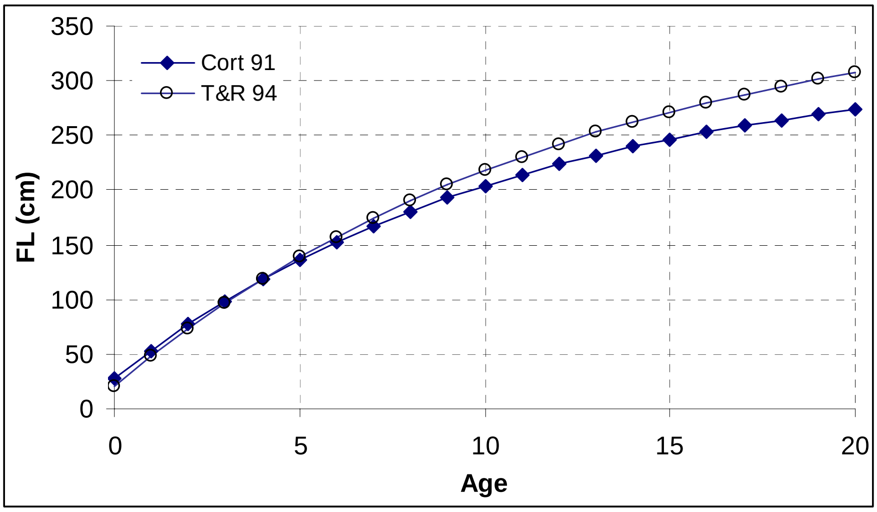


Figure 1. Growth curves for bluefin tuna currently used by SCRS for the eastern stock (from Cort, 1991) and for the western stock (from Turner and Restrepo, 1994).

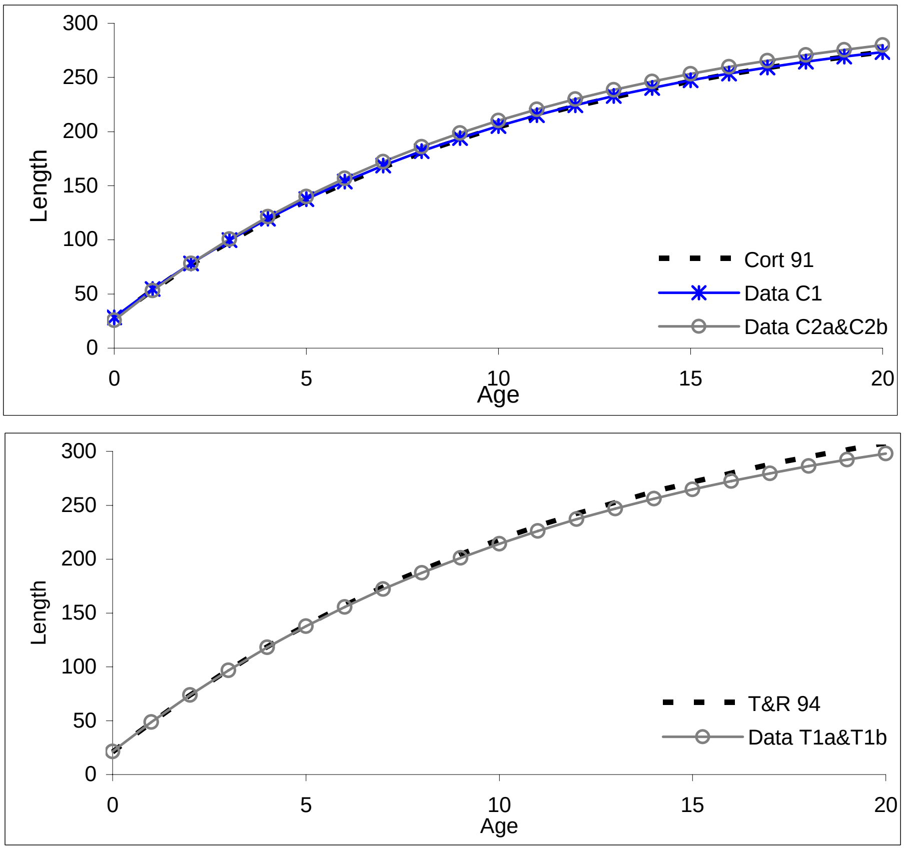
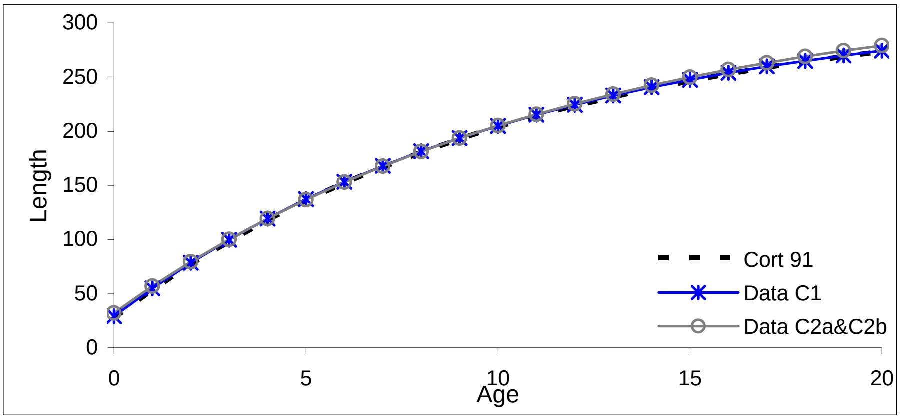


Figure 2a. Comparisons between the growth curves estimated by Cort (1991) and Turner and Restrepo ("T&R, 1994) and those derived from the reconstructed datasets in this study modeling age (time at liberty) as the dependent variable.



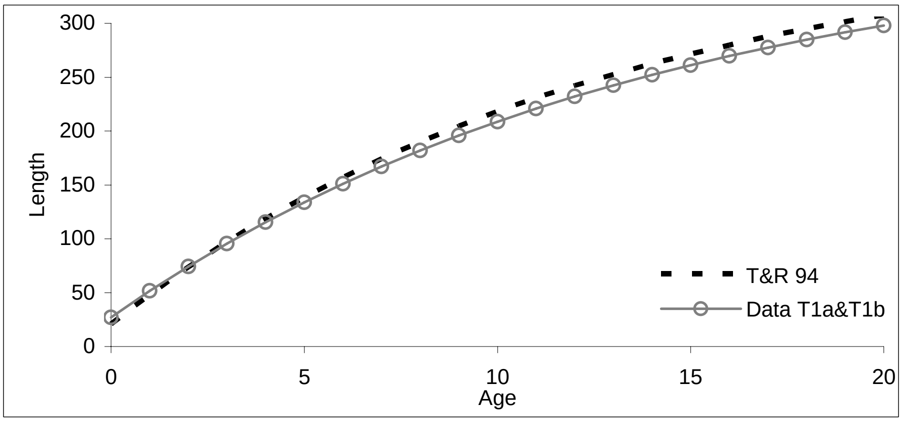


Figure 2b. Comparisons between the growth curves estimated by Cort (1991) and Turner and Restrepo ("T&R, 1994) and those derived from the reconstructed datasets in this study modeling length (length increment) as the dependent variable.

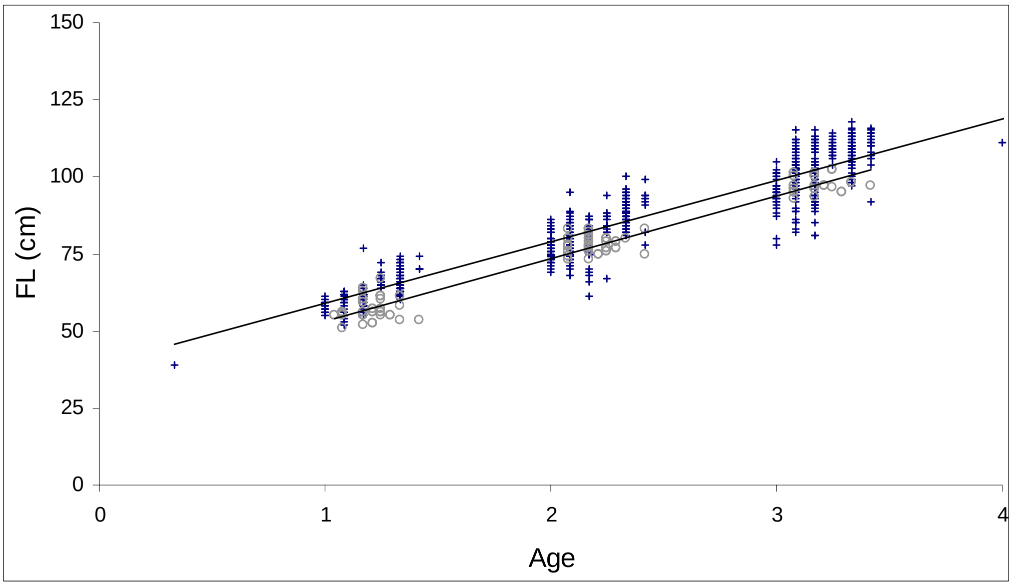


Figure 3. Length-age observations for datasets C2a (crosses) and T1b (circles), corresponding to small bluefin (only data up to age 4 are shown). The lines are linear trends fitted to each set.

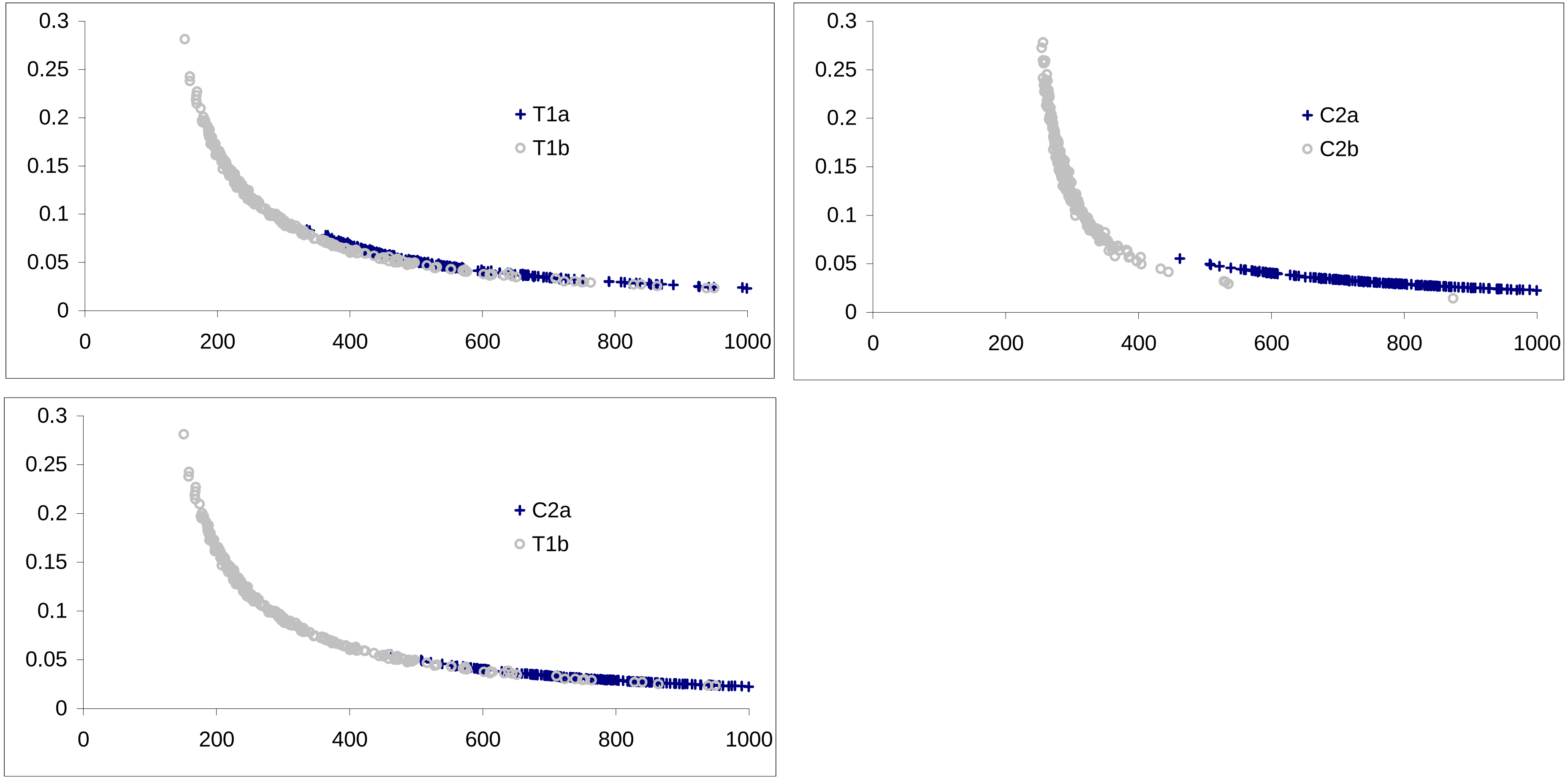


Figure 4. Bootstrap estimates of L_∞ (X-axis) and K (Y-axis) obtained modeling length as the dependent variable. Top left: sets T1a and T1b ; top right: sets C2a and C2b; bottom: sets C2a and T1b.